

Energy flow measurements in acoustic waves in a duct

Tetsushi Biwa *

Department of Mechanical Systems and Design, Tohoku University, Aoba-ku, Sendai, 980-0013, Japan

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Abstract

Where, how much and how efficiently the energy conversion takes place in a regenerator of a thermoacoustic engine are expressed using the axial distribution of acoustic work flow and heat flow. As a first step in determining the energy flows inside the regenerator, measuring methods of the work flow are briefly described and the experimental results in an acoustic resonator are shown. Applicability of these methods to the regenerator is discussed.

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1. Introduction

Coupled oscillations of pressure P , velocity U , and entropy s in an acoustic wave produce two energy flows; one is acoustic work flow $I = A \langle \overline{PU} \rangle$ and the other is heat flow $Q = T_m S$, where $S = A \rho_m \langle \overline{sU} \rangle$ is hydrodynamic entropy flow, ρ_m and T_m are time averages of density and temperature of the gas, A is the cross-sectional area of a flow channel and, bars and angular brackets mean the time and the radial averages of the quantity inside of them [1–4]. Furthermore, mutual conversions are possible between Q and I , when the acoustic wave travels in a small channel with an axial temperature gradient. Particularly when the channels are as narrow as those in a *regenerator*, the energy conversions are executed through a thermodynamic cycle similar to that in Stirling engines [5,6]. As a result, it has become possible to eliminate pistons from Stirling engines by using the thermoacoustic technique [7].

Now many thermoacoustic devices working as pistonless Stirling prime movers and coolers [8,9] have been proposed as an extremely simple and reliable technology for the energy industry. On the other hand, the fact that acoustic waves execute the energy conversions without pistons offers a new way to understand a heat engine, in which *heat*

flow Q and *work flow* I , instead of *heat* and *work*, constitute the basic concepts. In this paper, the energy conversions between Q and I are shown using energy flow diagrams in a thermoacoustic prime mover, and the experimental techniques for the measurement of I is introduced.

2. Energy flows in thermoacoustic devices

A thermoacoustic prime mover is schematically presented in Fig. 1a from the conventional view point of a heat engine. A regenerator is the heart of the prime mover, since the energy conversion takes place inside of it. The heat exchangers at the sides of the regenerator act as heat baths with temperatures T_H and T_C . Input heat power Q_H flows into the regenerator from the hot heat exchanger, while the heat power Q_C flows out of the regenerator from the cold heat exchanger. The difference between Q_H and Q_C equals the output power ΔI . The efficiency of the energy conversion is given as $\Delta I/Q_H$ and is always smaller than the Carnot efficiency $(1 - T_C/T_H)$, which is achieved only in an ideal regenerator without any irreversible processes.

The axial distribution of the heat flow Q and the work flow I in the regenerator are shown in Fig. 1(b). Here the positive value of the energy flow represents the flow to the right, and the negative one represents to the left. The magnitude of the heat flow Q decreases ($\nabla Q < 0$, where ∇Q means the divergence of Q) from Q_H to Q_C as it flows

* Fax: +81 52 789 3724.

E-mail address: biwa@amsd.mech.tohoku.ac.jp

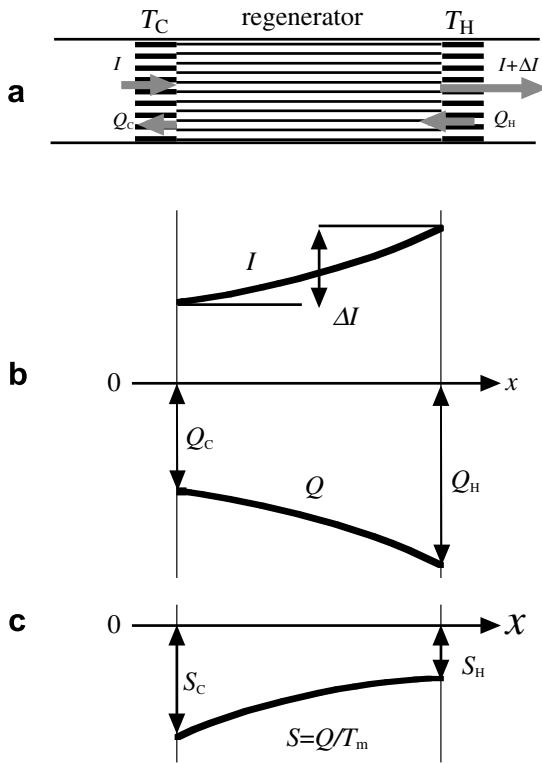


Fig. 1. Schematic illustration of a thermoacoustic heat engine (a) and axial distributions of acoustic work flow and heat flow (b) and entropy flow (c). A gray horizontal line in (c) represents the entropy flow in an ideal regenerator, and a gray curve in (b) is the corresponding heat flow in the ideal regenerator.

down the temperature gradient in the regenerator, while the work flow is amplified ($\nabla I > 0$) in the regenerator. From the first law of thermodynamics (energy conservation law), the relation

$$\nabla(Q + I) = 0 \quad (1)$$

always holds in the regenerator.

The work source $W \equiv \nabla I$ [3] represents the acoustic power production per unit length. In the conventional treatment of the heat engine, only the output power $\Delta I = Q_H - Q_C$ is discussed as a measure of the energy conversion, but from the thermoacoustic viewpoint, the measure of the energy conversion can be locally addressed by W . In the case of I shown in Fig. 1(b), we see that the rate of the energy conversion increases with increasing x .

In addition to W , the increase of the entropy flow S per unit length, ∇S , is also important in characterizing the local energy conversion in the regenerator. The entropy flow S is given by the heat flow Q divided by the local mean temperature T_m and is schematically shown in Fig. 1c. In the steady state, ∇S is equal to the entropy production per unit length and unit time. The second law of thermodynamics states $\nabla S = 0$ in the ideal regenerator, but otherwise $\nabla S > 0$ in actual regenerators with irreversible processes. Hence, ∇S can be used as a measure of how far the thermodynamic processes associated with the local energy conver-

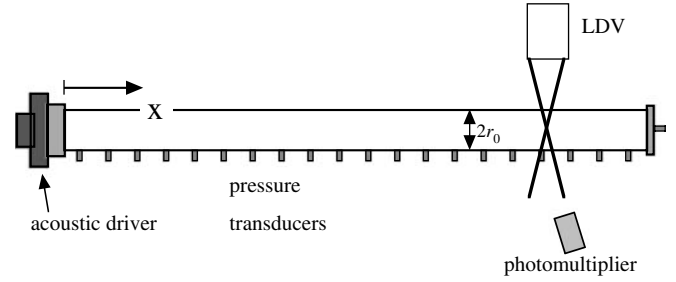


Fig. 2. Acoustic resonator driven by an acoustic driver. The origin of the axial coordinate x is taken at the front of the driver.

sion are away from reversible processes. In the case of S shown in Fig. 1(c), we see that the irreversible processes are more remarkable in the smaller x . Thus, we can see where, how much, and how efficiently the energy conversion takes place inside the regenerator from W and ∇S .

To obtain an experimental understanding of thermoacoustic energy conversions, it is essential to measure the axial distributions of Q and I , and determine W and ∇S . But the attempt to measure the energy flows has just begun. Measurement of the work flow I in a duct has recently been made possible, as will be introduced in the following section. The measurement of Q has not yet been established because of the difficulty of measuring the entropy oscillation s . Tashiro has succeeded in the measurements of the temperature oscillation T with a thermocouple [11]. By using this technique, he plans to determine the total energy flow $H = Q + I$, since H is given as $H = \rho C_p \langle T U \rangle$ for an ideal gas, where C_p denotes the isobaric specific heat of the gas. Acoustic heat flow Q can then be obtained as $Q = H - I$ from the measured I and H .

3. Work flow measurement

3.1. Experimental methods

Acoustic pressure $P(x) = p(x)\exp(i\omega t)$ with an angular frequency ω is a function of the axial coordinate x , but it is independent of the radial coordinate r in the tube, when the tube radius is much smaller than the wavelength. The axial acoustic velocity $U(x, r) = u(x, r)\exp[i\{\omega t + \phi(x, r)\}]$ depends on both x and r . Here the direction of the velocity is explicitly shown by its sign and we hereafter denote the velocity as U instead of U . By taking the radial average prior to the time average, the work flow $I(x) = \langle P(x)U(x, r) \rangle$ passing through the cross section at x is rewritten as

$$I(x) = \langle P(x)V(x) \rangle = \frac{1}{2}p(x)v(x)\cos\psi(x) \quad (2)$$

where $V(x) = v(x)\exp[i\{\omega t + \psi(x)\}]$ represents the radial average of the axial acoustic velocity. Therefore, the work flow I is derived from the measurements of P and V .

Yazaki and Tominaga [10] demonstrated the acoustic power generation associated with spontaneous gas oscillations from the measurements of the work flow I . They used

small pressure transducers for the measurement of the acoustic pressure, and a laser Doppler velocimeter (LDV) for the measurement of the axial velocity. This method was further applied to determine acoustic fields and acoustic output powers in thermoacoustic engines [7,9,12–14]. The quality factor of an acoustic resonator was also determined from the work flow measurements [15].

The measurement method of I which only involves pressure measurements has been reported by Fusco et al. [16]. In this method, P and V are calculated respectively from the sum and the difference of the pressure signals obtained by two adjacent pressure transducers. Simplicity of the measurements is a great advantage of this method, when compared to the method using the LDV. However, the estimated velocity and the work flow have not yet been verified yet by the simultaneous measurements of pressure and velocity. Next, we show how the work flow I can be determined in a simple acoustic resonator with these two methods in order to see the applicability of the latter method based on pressure measurements [17].

3.2. Work flow in an acoustic resonator

The experimental setup is shown in Fig. 2. An acoustic resonator was made of a Pyrex glass tube with length $L = 1.02$ m and inner radius $r_0 = 10.5$ mm, and filled with atmospheric air. One end of the resonator is closed by a rigid plate, and the other was connected to a compression driver.

The gas column in the resonator was driven at the frequency $f = 298.5$ Hz (the second mode), and the pressure amplitude at the closed end was kept at 1.00 kPa. The acoustic pressure $P(x) = p(x)\exp(i\omega t)$ was measured by a series of small pressure transducers mounted on the resonator wall via short ducts 10 mm in length. The axial acoustic velocity $U_c(x)$ on the central axis of the tube was measured using the LDV. Pressure and velocity signals were recorded with a multi-channel FFT analyzer. The measured velocity $U_c(x) = u_c(x)\exp[i\{\omega t + \phi_c(x)\}]$ was converted into the radial average $V(x) = v(x)\exp[i\{\omega t + \psi(x)\}]$ of the velocity using the theoretical solution of a laminar oscillating flow as

$$v(x) = u_c(x)/1.01, \quad (3A)$$

$$\psi(x) = \phi_c(x) + 0.7. \quad (3B)$$

The phase angles of $P(x)$ and $U_c(x)$ were accurately determined by taking into account delays due to electrical circuits and short ducts used in the pressure measurements.

Fig. 3 shows the axial distribution of the work flow $I(x)$ (open circles) derived from the simultaneous measurements of P and V . The measured I decreases from $I(0) = 24$ mW to zero as it flows to the rigid end. The total input power $I(0)$ from the compression driver gives the total energy loss E in the resonator. The experimental work source $W = -I(0)/L = -24$ W/m represents the dissipated power

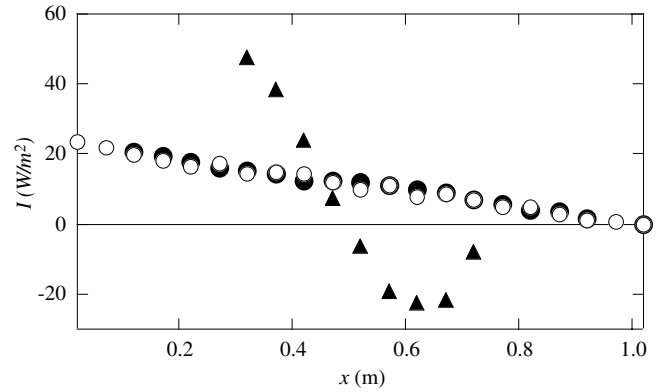


Fig. 3. Axial distribution of the work flow (open circles) derived from simultaneous measurements of P and V , (solid circles) from the pressure measurements with $X = 0.2$, and (solid triangles) with $X = 0.6$ m.

per unit length in the resonator. The acoustic energy E_S stored in the resonator was estimated as $E_S = 0.63$ mJ from the axial distribution of P and V , and we obtained the quality factor q of the resonator as $q = \omega E_S / E = 49$ [15]. This is in good agreement with the theoretical value of 55 derived from a linear acoustic theory.

3.3. Derivation of I from measured P

The acoustic pressure $P(x)$ and the radial average $V(x)$ of the velocity at a position x can be estimated [16] from the sum and the difference of pressures $P_A = P(x - X/2)$ and $P_B = P(x + X/2)$ measured with two pressure transducers separated by a distance X . Namely,

$$P(x) = \frac{P_A + P_B}{2 \cos(kX/2)} \quad (4)$$

$$V(x) = \frac{i}{\omega \rho} \left(1 - \frac{(1-i)\delta}{r_0} \right) \frac{(P_A - P_B)k}{2 \sin(kX/2)} \quad (5)$$

where

$$k = k_0 \left(1 + \frac{(1-i)\delta}{2r_0} + \frac{(1-i)\delta}{2r_0\sqrt{\sigma}} (\gamma - 1) \right), \quad (6)$$

$k_0 (= \omega/a, a$; adiabatic speed of sound) is the wave number in a free space, σ and γ are Prandtl number and the ratio of isobaric to isochoric specific heats of the gas, δ means the boundary layer thickness given as $\delta = \sqrt{2\nu/\omega}$ with using the kinematic viscosity ν . Eqs. (5) and (6) are valid when $r_0 \gg \delta$ (boundary-layer approximation). The work flow $I(x)$ is obtained by inserting $P(x)$ and $V(x)$ given by Eqs. (4) and (5) into (2).

The work flow $I(x)$ derived from the pressure data are shown as solid circles in Fig. 3. Here the distance X between the two measuring positions was chosen as $X = 0.2$ m. The relative error of the data obtained by pressure measurements was within 3%, and we conclude that this method is applicable to the resonator.

The pressure measurements in this method are simpler and easier than the velocity measurements with the LDV.

Furthermore, there is no difficulty in the pressure measurements in stainless-steel tubes filled with pressurized gases, whereas the LDV requires transparent tubes for the transmission of the laser light. High-power thermoacoustic engines are usually composed of stainless-steel tubes because pressurized gases are used as a working gas. Hence, the measuring method of I only from the pressure signals is most suitable to the study of the acoustic powers in such engines [17].

It should be noted here that the choice of the distance X is important to accurately obtain $I(x)$. We found significant error when X was taken as $X = 0.6$ m in the present resonator. The corresponding work flow $I(x)$ is shown in Fig. 3 as solid triangles. The large error originates from the fact that the denominator in Eq. (4) tends to zero when X approaches the half wavelength. Therefore, the choice of X close to the half-wavelength should be avoided for the accurate evaluation of $I(x)$ from the pressure measurements.

3.4. Applicability to the work flow in a regenerator

There are a few technical issues in determining of I from pressure signals in the regenerator. First, the validity of the boundary-layer approximation should be checked. Since viscous and thermal boundary layers of the gas fill up the channels in the regenerator, the boundary-layer approximation is no longer applicable. Secondly, a very high resolution is needed in the phase measurements. The length of the regenerator is usually much smaller than the wavelength, and hence the phase difference between two pressure signals is small. On the other hand, the relative error of I becomes large when the phase difference is small. Thus, the derivation of I using only the pressure signals is attractive, but challenging.

Technical issues in measuring I in a regenerator with the LDV are that the regenerator should be made of transparent materials and that its radius should be large enough compared to the size of the measuring volume of the LDV (typically, $\sim 0.1 \times 0.1 \times 1$ mm³). In order to resolve these issues, we have constructed a liquid-piston Stirling engine [18] having a regenerator made of a glass tube. The working frequency of a liquid-piston Stirling engine is as low as 1 Hz, and the thermal boundary layer thickness becomes a few millimeters long for one-bar air. As a result, we could enlarge the channel radius of the regenerator to 4 mm, which is much larger than those in other thermoacoustic engines. We confirmed the applicability of the velocity measurement in such a narrow tube from preliminary measurements [15]. Hence, we consider that the method with simultaneous measurements of P and V is more easily applied to the regenerator than that only with pressure measurements. The relatively large radius of the regenerator in the liquid-piston Stirling engine is also suitable for the temperature measurements using a thermocouple [11].

Measurements of P , V and T would be possible in the regenerator of the liquid-piston Stirling engine.

4. Summary

From the thermoacoustic viewpoint, the energy conversion *inside* the prime mover is expressed using the energy flows I and Q passing through the regenerator. Consequently, where, how much, and how efficiently the energy conversion takes place inside the regenerator are described by $W \equiv \nabla I$ and $\nabla S = \nabla(Q/T_m)$. This is in clear contrast to the conventional viewpoint in which the quantities only on the *outside* of the regenerator is discussed. In order to deepen our understanding of heat engines from the thermoacoustic viewpoint, measurements of I and Q in the regenerator is of great importance.

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